JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 17, No. 1, January–February 1994

Invertibility and Trajectory Control for Nonlinear Maneuvers of Aircraft

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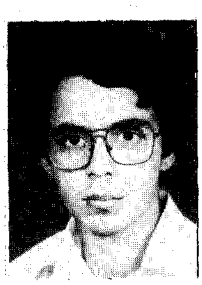
This paper presents an application of the inversion theory to the design of nonlinear control systems for simultaneous lateral and longitudinal maneuvers of aircraft. First, a control law for the inner loop is derived for the independent control of the angular velocity components of the aircraft along roll, pitch, and yaw axes using aileron, elevator, and rudder. Then by a judicious choice of angular velocity command signals, independent trajectory control of the sets of output variables (angle of attack, roll, and sideslip angles), (roll rate, angle of attack, and yaw angle), or (pitch, roll, and yaw angles) is accomplished. These angular velocity command signals are generated in the outer loops using state feedback and the reference angle of attack, pitch, yaw, and roll angle trajectories. Simulation results are presented to show that in the closed-loop system, various lateral and longitudinal maneuvers can be performed in spite of the presence of uncertainty in the stability derivatives.

I. Introduction

MANEUVERS such as rolling pullouts and high acceleration turns require simultaneous rapid responses in rolling and pitching. Such maneuvers are described by coupled nonlinear differential equations. Interesting results using bifurcation analysis and catastrophe theory have appeared in literature which indicate that for critical combinations of control inputs, divergence-like responses and limit cycles do appear.[1,2]

In recent years, an attempt has been made to design flight control systems for nonlinear maneuvers. Nonlinear decoupling theory[3,4] and a dynamic inversion approach have been applied to design control systems for aircraft.[5-10] Using the linearization theory by state feedback and nonlinear transformation, a flight control system has been designed in Ref. 11. Approximate tracking by inversion has been considered in Ref. 12.

This paper presents an application of nonlinear inversion theory[3,4] to the design of flight control systems. The contribution of this paper lies in the derivation of an inner control loop for the independent control of the angular velocity components p, q, and r, and the synthesis of outer control loops to generate command trajectories (p_c, q_c, r_c) to accomplish trajectory control of the sets of outputs [roll angle, angle of attack, and sideslip angle (ϕ, α, β)], [roll, pitch, and sideslip angle (ϕ, θ, β)], or [angular velocity p, angle of attack, and sideslip (p, α, β)] in the closed-loop system. Here, the objective is to accomplish various maneuvers by the suitable choice of the angular velocity command. This approach differs from the published works[5-7,10] where certain chosen outputs are directly controlled by the inputs. A related work[13] using inversion has come to the notice of the authors at the time of revision of this paper. However, the model of the aircraft and the control laws derived here are different. These command trajectories (p_c, q_c, r_c) are generated in the outer loops using nonlinear state feedback and the reference roll angle, angle of attack, sideslip angle, and pitch angle $(\phi_c, \alpha_c, \beta_c, \theta_c)$ trajectories. The advantage of the design of the inner (p, q, r) decoupled loop is



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Received Feb. 16, 1992; revision received Dec. 12, 1992; accepted for publication April 4, 1993.

that the outer loops are rather easily synthesized around the inner loop for accomplishing various simultaneous lateral and longitudinal maneuvers of aircraft. Extensive digital simulation results are presented to show simultaneous lateral and longitudinal maneuver capability of the controller in the presence of uncertainty in the stability derivatives.

II. Mathematical Model

The complete set of equations of motion of the rigid aircraft is given by (see Refs. 14 and 15 for the derivation and the notation)

$$\begin{bmatrix} \dot p \\ \dot q \\ \dot r \\ \dot\alpha \\ \dot\beta \\ \dot\phi \\ \dot\theta \end{bmatrix} = \begin{bmatrix} l_\beta\beta + l_q q + l_r r + (l_{\beta\alpha}\beta + l_{r\alpha} r)\Delta\alpha + l_p p - i_1 qr \\ \bar m_\alpha\Delta\alpha + \bar m_q q + i_2 pr - m_{\dot\alpha} p\beta + m_{\dot\alpha}(g/V)(\cos\theta\cos\phi - \cos\theta_0) \\ n_\beta\beta + n_r r + n_p p + n_{p\alpha} p\Delta\alpha - i_3 pq + n_q q \\ q - p\beta + z_\alpha\Delta\alpha + (g/V)(\cos\theta\cos\phi - \cos\theta_0) \\ y_\beta\beta + p(\sin\alpha_0 + \Delta\alpha) - r\cos\alpha_0 + (g/V)\cos\theta\sin\phi \\ p + q\tan\theta\sin\phi + r\tan\theta\cos\phi \\ q\cos\phi - r\sin\phi \end{bmatrix} + \begin{bmatrix} \tilde l_{\delta a} & l_{\delta r} & 0 \\ 0 & 0 & \bar m_{\delta e} \\ \tilde n_{\delta a} & n_{\delta r} & 0 \\ 0 & 0 & z_{\delta e} \\ y_{\delta a} & y_{\delta r} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta a \\ \delta r \\ \delta e \end{bmatrix} \triangleq f(x) + B(x)u \tag{1}$$

where $\Delta\alpha = \alpha - \alpha_0$, state vector $x = [p,q,r,\alpha,\beta,\phi,\theta]^T$, control vector $u = [\delta a,\delta r,\delta e]^T$, $\tilde l_{\delta a} = l_{\delta a} + l_{\alpha\delta a}\Delta\alpha$ and $\tilde n_{\delta a} = n_{\delta a} + n_{\alpha\delta a}\Delta\alpha$. Let

$$f(x) = [f_p(x), f_q(x), f_r(x), f_\alpha(x), f_\beta(x), f_\phi(x), f_\theta(x)]^T$$

where T denotes transposition. The primary output vector to be controlled is selected as

$$y = [p,q,r]^T \tag{2}$$

We are interested in deriving a control law such that in the closed-loop system each of the outputs can be independently controlled to follow angular velocity command trajectories p_c, q_c, r_c. Furthermore, analytical expressions for those angular velocity command trajectories as nonlinear functions of the state and the reference trajectories ϕ_c, α_c, β_c, and θ_c are derived for the independent trajectory tracking of (ϕ,α,β), (p,α,β), or (ϕ,θ,β).

III. Control of (p,q,r): Inner Loop Design

Invertibility of maps of nonlinear dynamical systems has been considered in Refs. 3, 4, and 16, and inversion algorithms have been presented. Using the inversion algorithm, one obtains a sequence of systems starting from the original system by the differentiation of the outputs and nonlinear transformations on the outputs. For the derivation of the control law, the inversion of the input-output map of the system relating $(\delta a,\delta r,\delta e)$ and (p,q,r) is considered.

Differentiating y and using Eq. (1) gives system 1

$$\begin{aligned} \dot x &= f(x) + B(x)u(t) \\ \dot y &= c_1(x) + D_1(x)u(t) \end{aligned} \tag{3}$$

where $c_1(x) = [f_p(x), f_q(x), f_r(x)]^T$ and the decoupling matrix is

$$D_1(x) = \begin{bmatrix} \tilde l_{\delta a} & l_{\delta r} & 0 \\ 0 & 0 & \bar m_{\delta e} \\ \tilde n_{\delta a} & n_{\delta r} & 0 \end{bmatrix} \tag{4}$$

The rank of $D_1(x)$ is 3 at each $x \in M$, where

$$M = \{x \in R^7 : (\tilde l_{\delta a} n_{\delta r} - \tilde n_{\delta a} l_{\delta r})\bar m_{\delta e} \neq 0\} \tag{5}$$

Thus, the (vector) relative degree of the system (1) and (2) is $(1,1,1)^3$; and the necessary and sufficient condition for the decoupling control of p, q, and r in the region M is satisfied.

We choose a control law of the form

$$u(t) = D_1^{-1}(x)[-c_1(x) + \dot y_c - \Lambda\tilde y] \tag{6}$$

where the command trajectory $y_c = (p_c,q_c,r_c)^T$, $\Lambda = \mathrm{diag}(\lambda_1,\lambda_2,\lambda_3)$, and the tracking error $\tilde y = (y - y_c)$. Substituting control law (6) in Eq. (3), gives

$$\dot{\tilde y} = -\Lambda\tilde y \tag{7}$$

In view of Eq. (7), for $\lambda_i > 0$, $i = 1,2,3$, it follows that $[\tilde p(t),\tilde q(t),\tilde r(t)] = [e^{-\lambda_1 t}\tilde p(0), e^{-\lambda_2 t}\tilde q(0), e^{-\lambda_3 t}\tilde r(0)]$ where $\tilde p(0) = p(0) - p_c(0)$, $\tilde q(0) = q(0) - q_c(0)$, $\tilde r(0) = r(0) - r_c(0)$, and $(\tilde p,\tilde q,\tilde r)^T = \tilde y$. Thus, $[p(t),q(t),r(t)]$ converges to $[p_c(t),q_c(t),r_c(t)]$ as $t \to \infty$. Moreover, if the initial tracking error $[\tilde p(0),\tilde q(0),\tilde r(0)]$ is zero, then $[p(t),q(t),r(t)] \equiv [p_c(t),q_c(t),r_c(t)]$ for all $t \geq 0$, and the command angular velocity trajectories are exactly reproduced in the closed-loop system. This completes the inner control loop design.

IV. Outer Feedback Loop Design

We consider now the derivation of command trajectories $p_c(t)$, $q_c(t)$, and $r_c(t)$ in the outer loop such that independent control of (ϕ,α,β), (ϕ,θ,β), or (p,α,β) can be accomplished.

A. (ϕ,α,β) Control

Suppose that a reference trajectory $z_{c1} = (\phi_c,\alpha_c,\beta_c)^T$ is given. It is desired to derive the command trajectories p_c, q_c, and r_c such that $z_1 = (\phi,\alpha,\beta)^T$ tracks z_{c1}^T.

To derive the expressions for $(p_c,\ q_c,\ r_c)$, let us consider the differential equations for ϕ, α, and β given in Eq. (1). We note that aileron, rudder, and elevator are principally moment producing devices, but they produce small forces as well since $y_{\delta a}$, $y_{\delta r}$, and $z_{\delta e}$ are nonzero. We assume that these small forces can be neglected, and thus we set $y_{\delta a} = y_{\delta r} = z_{\delta e} = 0$ in Eq. (5) for the control system design. However, nonzero values of these parameters are retained in the aircraft model for digital simulation. Under this assumption, one has

$$\begin{bmatrix} \dot\phi \\ \dot\alpha \\ \dot\beta \end{bmatrix} = \begin{bmatrix} 0 \\ z_\alpha\Delta\alpha + (g/V)(\cos\theta\cos\phi - \cos\theta_o) \\ y_\beta\beta + (g/V)\cos\theta\sin\phi \end{bmatrix} + \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ -\beta & 1 & 0 \\ (\sin\alpha_o + \Delta\alpha) & 0 & -\cos\alpha_o \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\triangleq h_1(\alpha,\theta,\beta,\phi) + G_1(\theta,\alpha,\phi,\beta)y \tag{8}$$

where h_1 and G_1 are defined in Eq. (8). The matrix G_1 is nonsingular at each $x \in M_1$, where

$$M_1 = \{x \in M : \cos\alpha_o + \tan\theta[(\sin\alpha_o + \Delta\alpha)\cos\phi + \beta\sin\phi\cos\alpha_o] \neq 0\} \tag{9}$$

We shall be interested in the trajectory of the system evolving on M_1. Since $\tilde{y} = y - y_c$, Eq. (8) gives

$$\dot{z}_1 = h_1(x) + G_1(x)(y_c + \tilde{y}) \tag{10}$$

Since in view of Eq. (7) $\tilde{y}$ tends to 0 when there is no uncertainty in the system parameters, Eq. (10) asymptotically becomes

$$\dot{z}_1 = h_1(x) + G_1(x)y_c \tag{11}$$

In view of Eq. (11), we select the command trajectory y_c such that

$$y_c = G_1^{-1}(x)[-h_1(x) + \dot{z}_{c1} - \mu_{11}\tilde{z}_1 - \mu_{10}\omega_1] \tag{12}$$

where constant parameters $\mu_{11}>0$, $\mu_{10}>0$, tracking error $\tilde{z}_1 = (z_1 - z_{c1}) = (\tilde{\phi},\tilde{\alpha},\tilde{\beta})^T = (\phi - \phi_c, \alpha - \alpha_c, \beta - \beta_c)^T$, and ω_1 is the integral of the tracking error satisfying

$$\dot{\omega}_1 = (\tilde{\phi},\tilde{\alpha},\tilde{\beta})^T \tag{13}$$

The integral feedback is introduced in Eq. (12) to obtain robustness in the control system to parameter uncertainty in the system. Substituting Eq. (12) in Eq. (10) gives

$$\dot{\tilde{z}}_1 = -\mu_{11}\tilde{z}_1 - \mu_{10}\omega_1 + G_1(x)\tilde{y} \tag{14}$$

Since $\tilde{y}(t)\rightarrow 0$ according to Eq. (7), Eq. (14) gives

$$\dot{\tilde{z}}_1 = -\mu_{11}\tilde{z}_1 - \mu_{10}\omega_1 \tag{15}$$

Differentiating Eq. (15) and using Eq. (13) gives

$$\ddot{\tilde{z}}_1 = -\mu_{11}\dot{\tilde{z}}_1 - \mu_{10}\tilde{z}_1 \tag{16}$$

The parameters μ_{ij} in Eq. (16) are chosen as $\mu_{11} = 2\zeta_1\omega_{n1}$, $\mu_{10} = \omega_{n1}^2$, $\zeta_1>0$, and $\omega_{n1}>0$ so that Eq. (16) is asymptotically stable. In the closed-loop system one has from Eq. (16) that $\tilde{z}_1(t) = (\phi - \phi_c,\ \alpha - \alpha_c,\ \beta - \beta_c)^T \rightarrow 0$, as $t\rightarrow\infty$ for $x(t) \in M_1$ and thus ϕ, α, and β follow the reference trajectories ϕ_c, α_c, and β_c in the closed-loop system. We also note from Eqs. (7) and (16) that $\tilde{y} \equiv 0$ and $\tilde{z}_1 \equiv 0$, provided that the initial conditions satisfy $\tilde{y}(0) = 0$, $\tilde{z}_1(0) = 0$, and $\omega_1(0) = 0$. The complete control law for ϕ, α, and β control is obtained by substituting $y_c = (p_c,q_c,r_c)^T$ from Eq. (12) and $\dot{y}_c$ in Eq. (6), where $\dot{y}_c$ is yet to be determined.

To this end, we introduce certain operators. The Lie derivative of a real valued function $\kappa(x)$ with respect to the vector field $f(x)$ is given by

$$L_f\kappa(x) = \left[\frac{\partial\kappa(x)}{\partial x}\right]f(x) \tag{17}$$

where $(\partial\kappa/\partial x)$ is a (1×7) row vector. For the vector function $h_1(x)$, we define

$$L_fh_1(x) = [L_fh_{11}(x), L_fh_{12}(x), L_fh_{13}(x)]^T \tag{18}$$

where $h_1(x) = [h_{11}(x), h_{12}(x), h_{13}(x)]^T$. For any matrix $M = [m_{ij}(x)]$, we define

$$L_fM(x) = [L_fm_{ij}(x)]$$

where $m_{ij}(x)$ denotes the element of M in the ith row and the jth column. Furthermore, the derivative of $G_1^{-1}(x)$ can be written as

$$\frac{d}{dt}G_1^{-1}(x) = -G_1^{-1}(x)\left[\frac{d}{dt}G_1(x)\right]G_1^{-1}(x) \tag{19}$$

which requires the computation of the derivative of $G_1(x)$. Since $G_1(x)$ and $h_1(x)$ are functions of only the variables α, θ, ϕ, and β, and the small control forces are neglected in controller design, one has

$$\begin{aligned}\frac{dh_1(x)}{dt} &= L_fh_1(x)\\ \frac{d}{dt}G_1^{-1}(x) &= -G_1^{-1}(x)[L_fG_1(x)]G_1^{-1}(x)\end{aligned} \tag{20}$$

Now differentiating y_c in Eq. (12) and using Eq. (20) gives

$$\begin{aligned}\dot{y}_c = &-G_1^{-1}(x)[L_fG_1(x)]G_1^{-1}(x)[-h_1(x) + \dot{z}_{c1} - \mu_{11}\tilde{z}_1 \\ &- \mu_{10}\omega_1] + G_1^{-1}(x)[-L_fh_1(x) + \ddot{z}_{c1} - \mu_{11}(L_fz_1 - \dot{z}_{c1}) \\ &- \mu_{10}\tilde{z}_1]\end{aligned} \tag{21}$$

The state variable feedback control law is obtained by substituting Eq. (21) for $\dot{y}_c$ in Eq. (6).

B. (ϕ,θ,β) Control

Let a reference trajectory $z_{c2}(t) = [\phi_c(t),\theta_c(t),\beta_c(t)]^T$ be given. We consider the derivation of the command trajectory (p_c,q_c,r_c) such that $z_2(t) = [\phi(t),\theta(t),\beta(t)]^T$ converges to $z_{c2}^T(t)$ as $t\rightarrow\infty$.

For the derivation of the command angular velocity trajectory, we set $(y_{\delta a} = y_{\delta r} = z_{\delta e} = 0)$ in Eq. (1). Then the differential equations for $z_2(t)$ is given by

$$\begin{aligned}\dot{z}_2 = &\begin{bmatrix} 0 \\ 0 \\ y_\beta\beta + (g/V)\cos\theta\sin\phi \end{bmatrix} \\ &+ \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ \sin\alpha_o + \Delta\alpha & 0 & -\cos\alpha_o \end{bmatrix} y \\ &\triangleq h_2(\beta,\theta,\phi) + G_2(\alpha,\theta,\phi)y\end{aligned} \tag{22}$$

where h_2 and G_2 are defined in Eq. (22). The matrix G_2 is nonsingular at each $x \in M_2$, where

$$M_2 = \{x \in M : \cos\phi\cos\alpha_o + \tan\theta(\sin\alpha_o + \Delta\alpha) \neq 0\} \tag{23}$$

We shall be interested in the control of (ϕ,θ,β) on M_2.

Since $\tilde{y} = y - y_c$, Eq. (22) gives

$$\dot{z}_2 = h_2(x) + G_2(x)(y_c + \tilde{y}) \tag{24}$$

Noting that $\tilde{y}(t)\rightarrow 0$ according to Eq. (7), in view of Eq. (24), we choose $y_c(t)$ of the form

$$y_c(t) = G_2^{-1}(x)[-h_2(x) + \dot{z}_{c2} - \mu_{21}\tilde{z}_2 - \mu_{20}\omega_2] \tag{25}$$

where $\tilde{z}_2 = (z_2 - z_{c2})^T = (\tilde{\phi},\tilde{\theta},\tilde{\beta})^T = (\phi - \phi_c, \theta - \theta_c, \beta - \beta_c)^T$, $\mu_{ij} > 0$, and

$$\dot{\omega}_2 = (\tilde{\phi},\tilde{\theta},\tilde{\beta})^T \tag{26}$$

Substituting $y_c(t)$ from Eq. (25) in Eq. (24), we get

$$\dot{\tilde{z}}_2 = -\mu_{21}\tilde{z}_2 - \mu_{20}\omega_2 + G_2(x)\tilde{y} \tag{27}$$

Since in view of Eq. (7), $\tilde{y}(t)\to 0$, as $t\to\infty$, asymptotically Eq. (27) satisfies

$$\dot{\tilde{z}}_2 = -\mu_{21}\tilde{z}_2 - \mu_{20}\omega_2 \tag{28}$$

Differentiating Eq. (28) and using $\dot{\omega}_2$ from Eq. (26) gives

$$\ddot{\tilde{z}}_2 = -\mu_{21}\dot{\tilde{z}}_2 - \mu_{20}\tilde{z}_2 \tag{29}$$

For a choice of $\mu_{21} = 2\zeta_2\omega_{n2}$, $\mu_{20} = \omega_{n2}^2$, $\zeta_2 > 0$, $\omega_{n2} > 0$, it follows from Eq. (29) that $\tilde{z}_2 \to 0$ as $t\to\infty$. Furthermore, in view of Eqs. (7), (26), and (27), if the initial conditions satisfy, $\tilde{y}(0) = 0$, $\tilde{z}_2(0) = 0$, and $\omega_2(0) = 0$, one has $\phi(t) \equiv \phi_c(t)$, $\theta(t) \equiv \theta_c(t)$, and $\beta(t) \equiv \beta_c(t)$, for all $t \geq 0$ and roll, pitch, and yaw angles exactly follow the prescribed trajectories $\phi_c(t)$, $\theta_c(t)$, and $\beta_c(t)$.

The feedback control law for ϕ, θ, and β control is given by Eqs. (6) and (25). Differentiating Eq. (25), using Eq. (1) and neglecting small control forces, we get

$$\begin{aligned}\dot{y}_c = {} & -G_2^{-1}(x)[L_f G_2(x)]G_2^{-1}(x)[-h_2(x) + \dot{z}_{c2} - \mu_{21}\tilde{z}_2 \\ & - \mu_{20}\omega_2] + G_2^{-1}(x)[-L_f h_2(x) + \ddot{z}_{c2} - \mu_{21}(L_f z_2 - \dot{z}_{c2}) \\ & - \mu_{20}\tilde{z}_2]\end{aligned} \tag{30}$$

Substituting y_c and $\dot{y}_c$ from Eqs. (25) and (30) in Eq. (6) gives the complete control law for ϕ, θ, and β control.

C. (p,α,β) Control

Here the objective is to derive the angular velocity command such that given reference trajectories $p_c(t)$, $\alpha_c(t)$, and $\beta_c(t)$ are tracked. Using the control law (6), one can follow any given $p_c(t)$. However, to obtain robustness, we introduce an integral term for p control. The modified control law obtained from Eq. (6) is

$$u(t) = D_1^{-1}(x)\left[-c_1(x) + \dot{y}_c - \Lambda\tilde{y} - \left(\lambda_o \int_0^t \tilde{p}\, \mathrm{d}t, 0, 0\right)^T\right] \tag{31}$$

where the diagonal matrix Λ is such that $\lambda_1 = 2\zeta_o\omega_{no}$, $\lambda_o = \omega_{no}^2$, $\zeta_o > 0$, $\lambda_2 > 0$, and $\lambda_3 > 0$. Substituting Eq. (31) in Eq. (3) we get

$$\begin{pmatrix}\dot{\tilde{p}}\\ \dot{\tilde{q}}\\ \dot{\tilde{r}}\end{pmatrix} = \begin{pmatrix}-\lambda_1\tilde{p} - \lambda_o\int_{0}^{t}\tilde{p}\,\mathrm{d}t\\ -\lambda_2\tilde{q}\\ -\lambda_3\tilde{r}\end{pmatrix} \tag{32}$$

We note from Eq. (32) that tracking error $\tilde{p}$ satisfies

$$\ddot{\tilde{p}} = -\lambda_1\dot{\tilde{p}} - \lambda_o\tilde{p}$$

For the chosen λ_i parameters, we note from Eq. (32) that $[\tilde{p}(t),\tilde{q}(t),\tilde{r}(t)]\to 0$, as $t\to\infty$.

The command angular velocity $p_c(t)$ is already specified. Now we derive expressions for the command trajectories $q_c(t)$ and $r_c(t)$ so that in the closed-loop system the reference trajectories $\alpha_c(t)$ and $\beta_c(t)$ can be followed.

We consider the differential equations for α and β obtained from Eq. (1) by neglecting small control forces ($y_{\delta a} = y_{\delta r} = z_{\delta e} = 0$). These are

$$\dot{z}_3 = \begin{pmatrix} z_\alpha\Delta\alpha + (g/V)(\cos\theta\cos\phi - \cos\theta_o) - p\beta \\ y_\beta\beta + (g/V)\cos\theta\sin\phi + p(\sin\alpha_o + \Delta\alpha)\end{pmatrix} + \begin{pmatrix} q \\ -r\cos\alpha_o\end{pmatrix} \triangleq h_3(\alpha,\beta,p,\theta,\phi) + G_3(q,r)^T \tag{33}$$

where h_3 and G_3 are defined in Eq. (33), and $z_3 = (\alpha,\beta)^T$. We note that G_3 is nonsingular on M. Since $(q,r) = (q_c,r_c) + (\tilde{q},\tilde{r})$, Eq. (33) gives

$$\dot{z}_3 = h_3(x) + G_3[(q_c,r_c) + (\tilde{q},\tilde{r})]^T \tag{34}$$

Since $(\tilde{p},\tilde{q},\tilde{r})\to 0$ and $p\to p_c$, as $t\to\infty$, according to Eq. (32) for any given (p_c,q_c,r_c), one obtains the desired command trajectory (q_c,r_c) in view of Eq. (34) of the form

$$\begin{pmatrix} q_c \\ r_c\end{pmatrix} = G_3^{-1}[-h_3(\alpha,\beta,\theta,\phi,p_c) + \dot{z}_{c3} - \mu_{31}\tilde{z}_3 - \mu_{30}\omega_3] \tag{35}$$

$$\dot{\omega}_3 = (\tilde{\alpha},\tilde{\beta})^T$$

where the reference trajectory $z_{c3} = (\alpha_c,\beta_c)^T$, $\mu_{31} = 2\zeta_3\omega_{n3}$, $\mu_{30} = \omega_{n3}^2$, $\zeta_3 > 0$, and $\omega_{n3} > 0$. Here p_c has been substituted for p in h_3.

Substituting Eq. (35) in Eq. (34), we get

$$\dot{\tilde{z}}_3 = (\dot{\tilde{\alpha}},\dot{\tilde{\beta}})^T = -\mu_{31}\tilde{z}_3 - \mu_{30}\omega_3 + G_3(\tilde{q},\tilde{r})^T \tag{36}$$

Since $(\tilde{q},\tilde{r})\to 0$, as $t\to\infty$ according to Eq. (32), Eq. (36) gives

$$\dot{\tilde{z}}_3 = -\mu_{31}\tilde{z}_3 - \mu_{30}\omega_3 \tag{37}$$

Differentiating Eq. (37) and using Eq. (35) gives

$$\ddot{\tilde{z}}_3 = -\mu_{31}\dot{\tilde{z}}_3 - \mu_{30}\tilde{z}_3 \tag{38}$$

Since Eq. (38) is asymptotically stable, $[(\alpha - \alpha_c),(\beta - \beta_c)]\to 0$, as $t\to\infty$. Furthermore, if $\tilde{y}(0) = 0$, $\tilde{z}_3(0) = 0$, $\omega_3(0) = 0$, $\alpha(t) \equiv \alpha_c(t)$, $\beta(t) \equiv \beta_c(t)$, and $p(t) \equiv p_c(t)$ for all $t \geq 0$.

The control law for (p,α,β) control is given by Eqs. (31) and (35). Here p_c is set equal to the reference p-trajectory to be

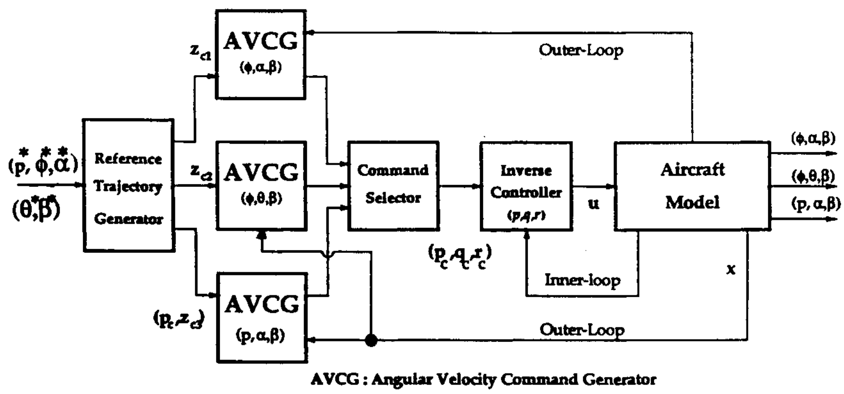
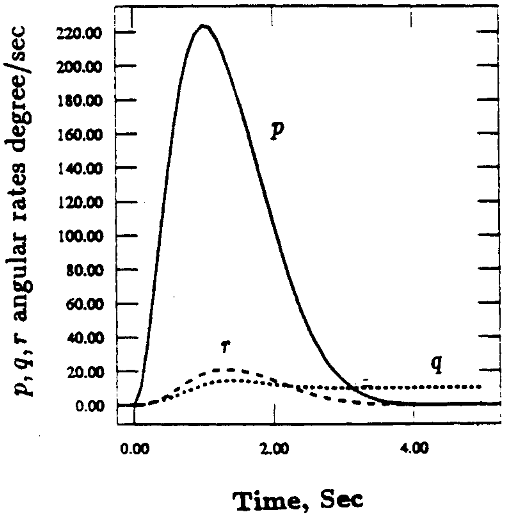


Fig. 1 Closed-loop system.



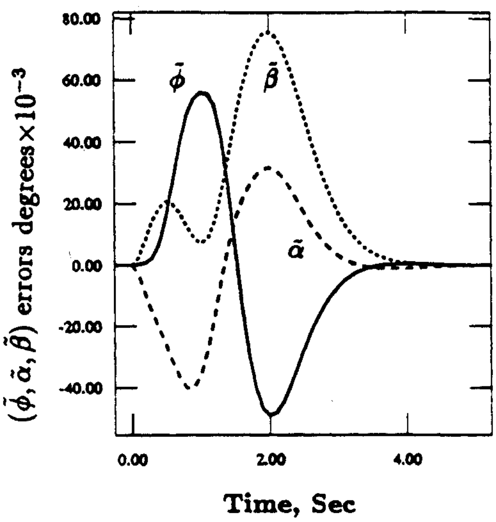
a) p,q,r angular rates

tracked. The derivatives of (q_c, r_c) for substitution in Eq. (31) are obtained by differentiating Eq. (35) and are

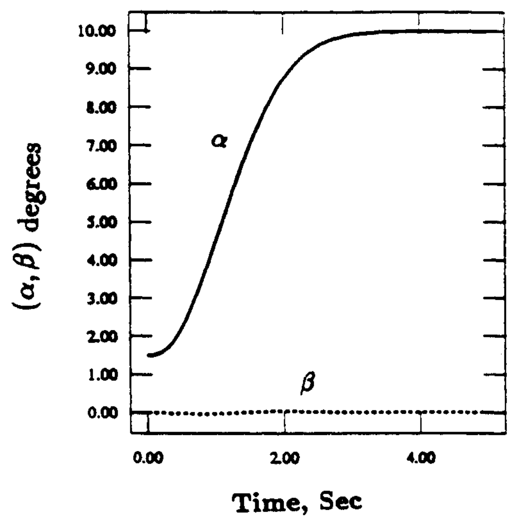
$$\begin{pmatrix}\dot q_c\\ \dot r_c\end{pmatrix} = G_3^{-1}[-L_f h_3(x,p_c) + (\partial h_3/\partial p_c)\dot p_c] + \ddot z_{c3} - \mu_{31}(L_f z_3 - \dot z_{c3}) - \mu_{30}\tilde z_3 \tag{39}$$

Using Eq. (39) in Eq. (31) gives the state variable feedback control law for (p,α,β) tracking.

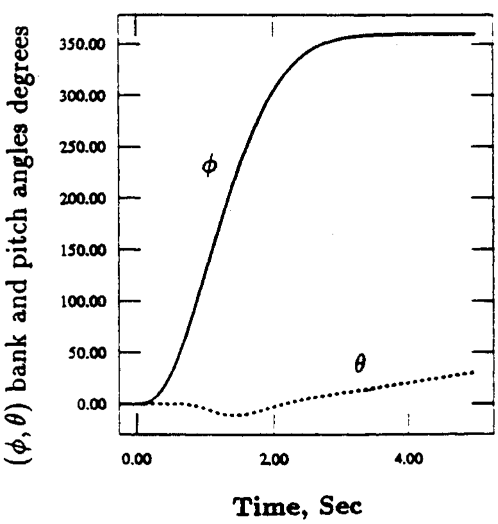
The complete closed-loop system including the inner loop and the outer loops generating the command angular velocity vector (p_c,q_c,r_c) for (ϕ,α,β), (p,α,β), and (ϕ,θ,β) control is shown in Fig. 1. We observe that the inner loop is for controlling the angular velocity components. The outer loops generate the command (p_c,q_c,r_c) so that different maneuvers can be performed. The required maneuver is accomplished by selecting the corresponding angular velocity command using the command selector.



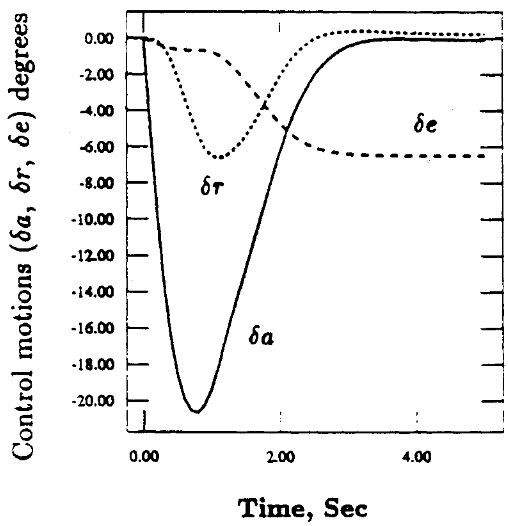
b) $(\tilde\phi,\tilde\alpha,\tilde\beta)$ errors



d) (α,β) angle of attack and sideslip angle



c) (ϕ,θ) bank and pitch angles



e) Control motions

Fig. 2 Control of ϕ,α,β, nominal case.

V. Numerical Results

In this section, simulation results are presented for the aircraft model studied in Refs. 14 and 15 for the two flight conditions, namely for condition I: $M = 0.9$ and $h = 20{,}000$ ft, and for condition II: $M = 0.7$ and $h = 0$. The complete set of aerodynamic parameters is given in Ref. 15, and we have $\alpha_0 = 1.5$ deg, $\theta_0 = 0$, and $y_{\delta r} = 0$. The nonzero parameters $y_{\delta a}$ and $z_{\delta e}$ given in Ref. 15 are introduced in the model of the aircraft for simulation to include the effect of control forces. For the purpose of illustration, it is convenient to use third-order filters for generating smooth commands since second-order derivatives of command trajectories are used in the control laws. The command generators are chosen of the form

$$\Pi(s)[\gamma_c(t) - \gamma^*] = 0$$

where $s = \mathrm{d}/\mathrm{d}t$; $\gamma^* = (\phi^*,\alpha^*,\beta^*,\theta^*)^T$ denotes the terminal value of the variables, $\gamma_c = (\phi_c,\alpha_c,\beta_c,\theta_c)^T$, and the polynomial $\Pi(s)$ is given by

$$\Pi(s) = (s + \lambda_c)(s^2 + 2\zeta_c\omega_{nc}s + \omega_{nc}^2) \tag{40}$$

Here $\omega_{nc} = \lambda_c/\zeta_c$ and $\zeta_c = 0.707$. The initial conditions chosen for the filter are $\phi_c(0) = \dot\phi_c(0) = \ddot\phi_c(0) = 0$, $\alpha_c(0) = 1.5$ deg, $\dot\alpha_c(0) = \ddot\alpha_c(0) = 0$, $\beta_c(0) = \dot\beta_c(0) = \ddot\beta_c(0) = 0$, $\theta_c(0) = \dot\theta_c(0) = \ddot\theta_c(0) = 0$, and $\omega_i(0) = 0$ $(i = 1,2,3)$. The value of parameter Λ in Eq. (6) was set to $\Lambda = \mathrm{diag}(5,5,5)$ giving a time constant of 0.2 for the angular velocity tracking error responses. The feedback gains μ_{ij} chosen are $\mu_{i1} = 2 \times 0.707 \times \omega_{ne}$, $\mu_{i0} = \omega_{ne}^2$ $(i = 1,2,3)$, $\omega_{ne} = 9$ giving pole locations associated with the tracking error responses for $\tilde z_1$, $\tilde z_2$, and $\tilde z_3$ in Eqs. (16), (29), and (38), respectively, at $-6.363 \pm j6.365$. Thus, the trajectory of $\tilde\phi$, $\tilde\alpha$, $\tilde\beta$, and $\tilde\theta$ will have similar convergence property. These feedback parameters were selected by examining the simulated responses.

A. Control of ϕ, α, β

1. *Nominal Control*

Responses were obtained to examine (ϕ,α,β) tracking ability using the parameters of the aircraft at flight condition I with the initial condition $x(0) = 0$ [except $\alpha(0) = 1.5$ deg] using the control law (6) and Eq. (12). The terminal values of ϕ, α, and β were chosen as $(\phi^*,\alpha^*,\beta^*) = (360,\ 10,\ 0$ deg). The filter parameter was set to $\lambda_c = 1.5$. Simulated responses are shown in Fig. 2. We observe smooth tracking of the reference trajectories ϕ_c, α_c, and β_c. The response time was of the order of 3 s. The tracking error $(\tilde\phi,\tilde\alpha,\tilde\beta)$ was less than (0.06, 0.08, 0.04 deg). The maximum value of control input $(\delta a,\delta r,\delta e)$ was about (20, 6, 7 deg). Magnitude of control input can be reduced by choosing slower command inputs. However, this

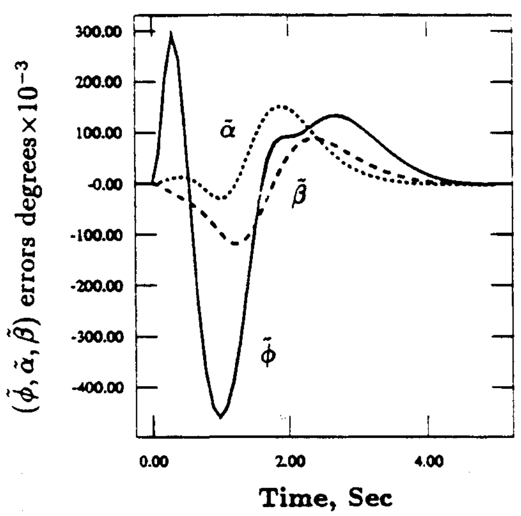
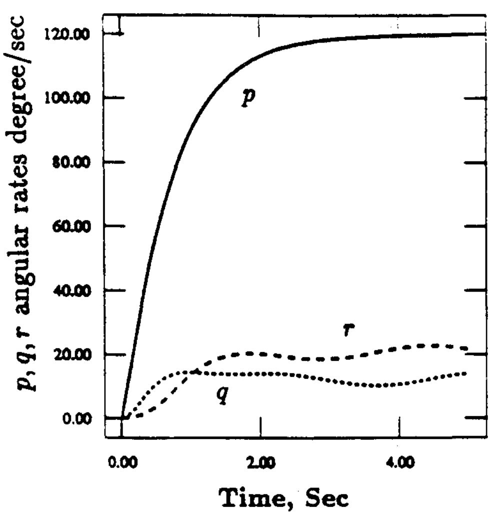


Fig. 3 Control of ϕ,α,β, off-nominal flight condition II: $(\tilde\phi,\tilde\alpha,\tilde\beta)$ errors.



a) p,q,r angular rates

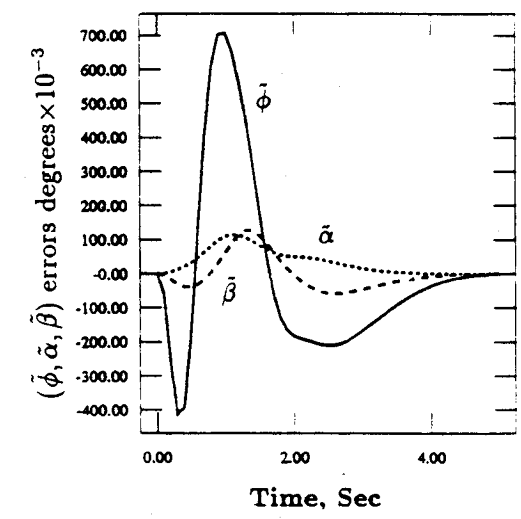
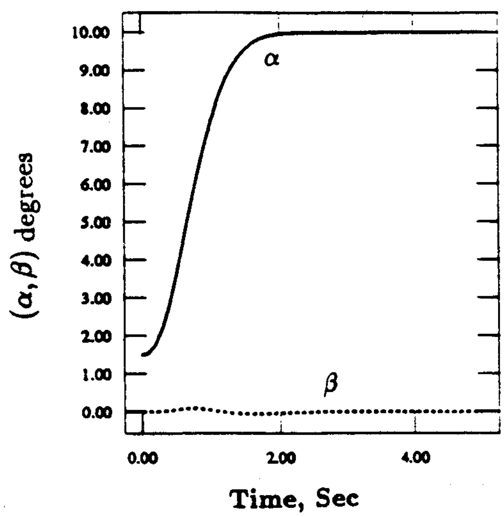


Fig. 4 Control of ϕ,α,β, off-nominal flight condition I: $(\tilde\phi,\tilde\alpha,\tilde\beta)$ errors.



b) (α,β) angle of attack and sideslip angle

Fig. 5 Control of p,α,β, nominal case.

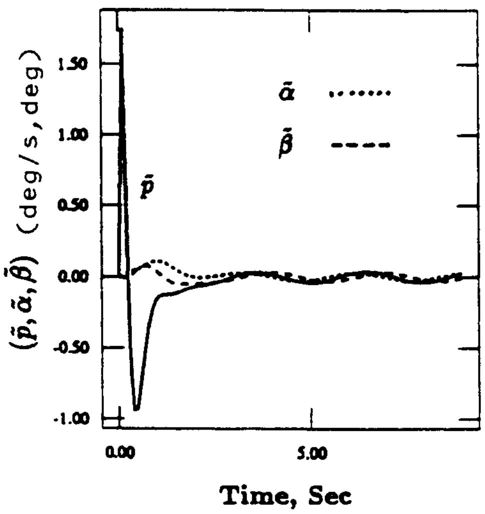
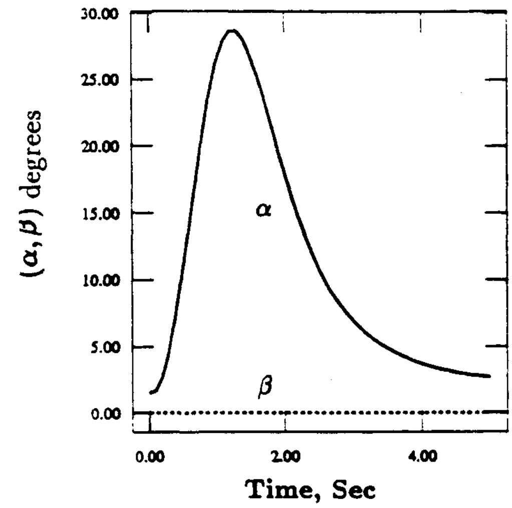


Fig. 6 Control of p,α,β, off-nominal flight condition II: $(\tilde{p},\tilde{\alpha},\tilde{\beta})$ errors.



a) (α,β) angle of attack and sideslip angle

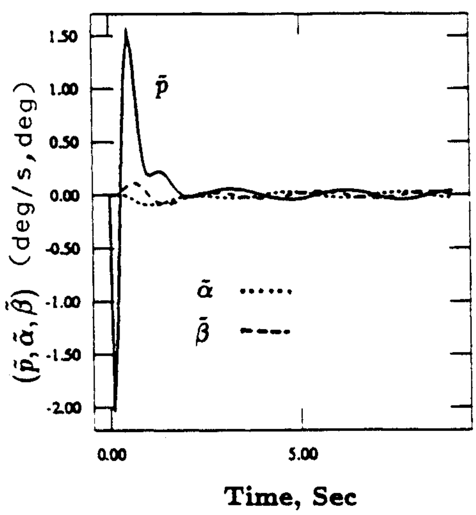
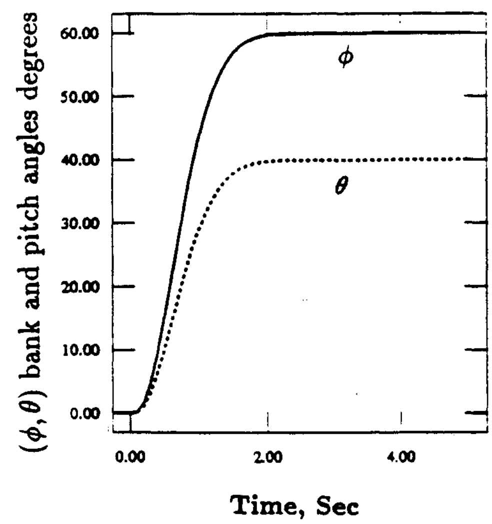


Fig. 7 Control of p,α,β, off-nominal flight condition I: $(\tilde{p},\tilde{\alpha},\tilde{\beta})$ errors.



b) (ϕ,θ) bank and pitch angles

Fig. 8 Control of ϕ,θ,β, nominal case.

choice will require longer response time to attain the final value.

2. Perturbed Aircraft Parameters of Flight Condition II

To examine the sensitivity of the controller, the aerodynamic coefficients of the aircraft were taken to be of flight condition II in simulation. However, controller designed for flight condition I of the case in Sec. V.A.1. was retained. The remaining parameters and the initial conditions of the case in Sec. V.A.1. were kept for simulation. Selected responses are shown in Fig. 3. Responses were very similar to those in Sec. V.A.1. In spite of the perturbed parameters of the aircraft, smooth responses were obtained. Although oscillatory tracking error responses are obtained for the chosen feedback functions, convergent output trajectories are observed. The maximum value of control input is less than (25, 8, 5 deg).

3. Perturbed Aircraft Parameters of Flight Condition I

The controller was designed using the aerodynamic parameters at flight condition II, but in simulation the aerodynamic coefficients of the aircraft were taken to be of flight condition I. Selected responses are shown in Fig. 4. The remaining responses were somewhat similar to those in Sec. V.A.1. The maximum value of control input is less than (22, 6, 4 deg).

B. Control of p, α, β

1. Nominal Control

Control of p, α, and β at flight condition I using control law (31) and Eq. (35) was considered. The controller gains μ_{ij}, λ_2, and λ_3 of Sec.V.A.1. were retained. The parameters $\lambda_1 = 12.726$ and $\lambda_o = \omega_{ne}^2$ were assumed. The reference trajectories α_c and β_c were generated using filter (40) with $\lambda_c = 2.5$. For illustration, a reference trajectory of the form $p_c(t) = p^*(1 - e^{-1.5t})$, with $p^* = 120$ deg/s was selected. The terminal values of α and β were taken as $\alpha^* = 10$ deg and $\beta^* = 0$ deg. Selected simulated responses are shown in Fig. 5. The reference trajectories were smoothly followed. The maximum tracking error for $(\tilde{\alpha},\tilde{\beta})$ was less than (0.04, 0.09 deg), and $\tilde{p}$ was zero. The maximum control magnitude was about $(\delta a, \delta r, \delta e) = (11, 6, 6$ deg).

2. Perturbed Aircraft Parameters of Flight Condition II

The controller designed of case Sec. V.B.1. was retained, but the perturbed parameters of flight condition II were used for the aircraft model in simulation. The reference trajectory for $p_c(t)$ of Sec.V.B.1. was retained. In spite of the off-nominal condition, smooth responses were obtained. We observed small tracking error. This is caused by continuous rolling, which introduces roll-coupled nonlinear terms in the equations of motion. However, no significant effect on the responses of the closed-loop system is observed. Selected results are shown in Fig. 6. The remaining responses have much similarity to those in Sec.V.B.1. The maximum tracking error for $(\tilde{\alpha},\tilde{\beta})$ was less than (0.12, 0.08 deg) and $\tilde{p}$ is 1.8 deg/s. We observe only a small steady-state error $(\tilde{p},\tilde{\alpha},\tilde{\beta}) \cong$ (0.038 deg/s, 0.029 deg, 0.0306 deg) after 5 s. The maximum control magnitude was $(\delta a, \delta r, \delta e)$ and was less than (12, 6, 3 deg).

3. Perturbed Aircraft Parameters of Flight Condition I

Simulation was also done using controller designed for flight condition II, but aircraft parameters were taken to be of flight condition I. For the same reference trajectory given in Sec.V.B.1., the simulation results were obtained. These results are shown in Fig. 7. Responses were somewhat similar to those in Sec.V.B.1. As in Sec.V.B.2., oscillatory tracking error of insignificant magnitude was observed.

C. Control of ϕ, θ, β

1. Nominal Control

Responses were obtained to examine the tracking ability of the variables ϕ, θ, and β at flight condition I using the control law (6) and Eq. (25). The controller parameters of Sec.V.A.1. were retained. The reference trajectory ϕ_c, θ_c, and β_c were generated using the filter (40) with $\lambda_c = 2.5$. The terminal values of $\phi^* = 60$ deg, $\theta^* = 40$ deg, and $\beta^* = 0$ deg were assumed. The smooth convergence of reference trajectories was obtained. Selected simulated responses are shown in Fig. 8. The maximum control input u was about (6, 11, 21 deg).

2. Perturbed Aircraft Parameters of Flight Condition II

The controller designed for Sec.V.C.1. was retained, but the perturbed parameters of flight condition II were used for the aircraft model in simulation. In spite of the off-nominal condition, smooth responses were obtained. Convergent output trajectories were observed. The reference trajectories were smoothly followed. Simulation results are shown in Fig. 9. The responses that are more or less similar to those of Sec.V.C.1. are not shown here. The maximum tracking error for $(\tilde{\phi},\tilde{\theta},\tilde{\beta})$ was less than (0.8, 0.6, 0.1 deg). The maximum control magnitude $(\delta a, \delta r, \delta e)$ was about (5, 8, 7 deg).

3. Perturbed Aircraft Parameters of Flight Condition I

Simulation was also done using a controller designed for flight condition II, but aircraft parameters were taken to be of flight condition I. Selected results are shown in Fig. 10. Convergent output trajectories are observed.

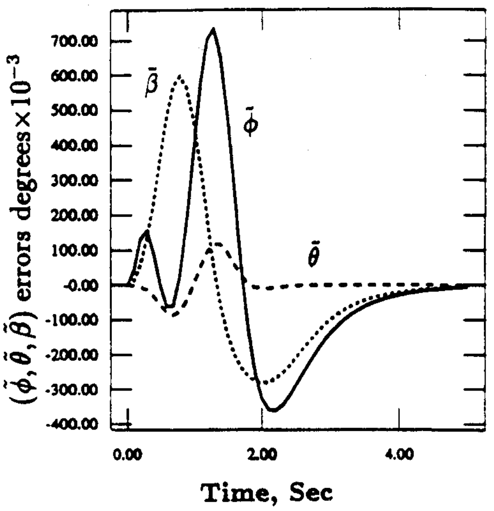


Fig. 9 Control of ϕ,θ,β, off-nominal flight condition II: $(\tilde{\phi},\tilde{\theta},\tilde{\beta})$ errors.

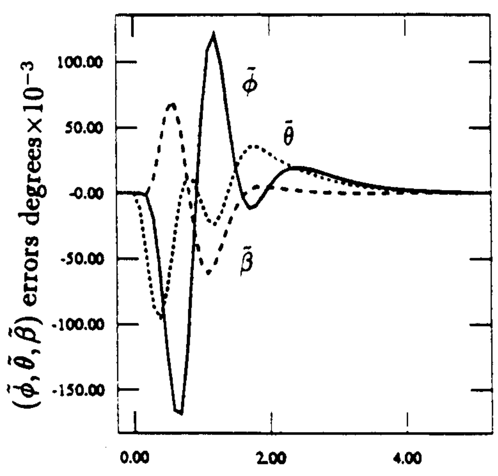


Fig. 10 Control of ϕ,θ,β, off-nominal flight condition I: $(\tilde{\phi},\tilde{\theta},\tilde{\beta})$ errors.

VI. Conclusions

Based on nonlinear inversion theory, a design approach for nonlinear maneuvers of aircraft was presented. The control system consists of an inner loop for the trajectory control of (p,q,r). Outer loops were designed around the decoupled inner loop for the control of (ϕ,α,β), (ϕ,θ,β), and (p,α,β). Analytical expressions for angular velocity command signals were derived to obtain desired maneuvers in the closed-loop system. These command signals are nonlinear functions of the state variables and the reference trajectories to be tracked in the closed-loop system. Simulation results were presented to show that in the closed-loop system, simultaneous longitudinal and lateral maneuvers can be performed in spite of the uncertainty in the stability derivatives.

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